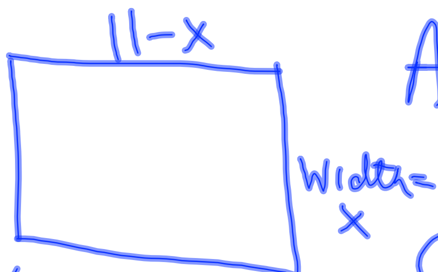
Day 4: Modeling Real Situations using Quadratic Functions

Suppose you have 22m of fencing that will be used to build a rectangular pen. What should the dimensions of the pen be so that the area of the pen is a maximum?

There are 2 ways to solve this problem:
Guess and check...The wrong way!!
Quadratic Equations...The right way!

Solve the problem by first writing expressions for length and width to get an expression for area. Your expression for area will be a quadratic, which should generate a graph with a maximum point.



$P=2l+2w$

$22=2l+2w$

$11=l+w$

$11=l+x$

$11-x=l$

$A=l \times w$ *

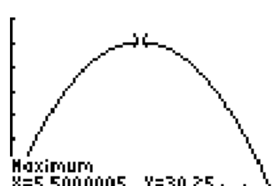
$A=(11-x)x$

$A=11x-x^2$

X	Y1
5	30
6	30
7	28
8	24
9	18
10	10
11	0

X=11

x:[0, 11, 1]
y:[0, 35, 5]



width=5.5

length = 11 − x width

= 11 − 5.5

= 5.5

max area

Example 2: A small fitness club has 160 members who each pay \$400 per year in membership fees. The manager estimates that for every \$25 decrease in the yearly membership fee an additional 20 members will join the club.

a) Represent the number of members as a function of the yearly membership.

Write an eqn for the # of members.

let n = # of members

let f = yearly fee.

$n = \left(\text{current members}\right) + \left(\text{new members}\right)$

$n = 160 + 20\left(\frac{400-f}{25}\right)$

$n = 160 + \frac{8000-20f}{25}$

$n = 160 + 320 - 0.8f$

$n = 480 - 0.8f$

b) Represent the revenue as a function of the membership fee

$R = (\text{fee}) * (\#\text{ of members})$

$R = f(480 - 0.8f)$

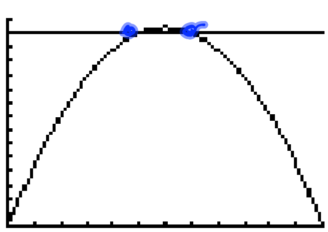
$R = 480f - 0.8f^2$

c) Graph the function and determine the which membership fee will provide the maximum revenue.

x: [0, 600, 50]

Y: [0, 75000, 5000]

d) What range of membership fees will provide revenue greater than \$70000?



Between \$250 and \$350.

New members

$20\left(\frac{\#400-f}{25}\right)$

every decrease of $25 increase of 20 members.

$20\cdot\left(\frac{400-350}{25}\right)$

$400-350$

$\frac{50}{25}=2$

of $25 decreases

2×20

40

<u>Example 3:</u> A bus company currently charges a fare of $50 on one of its routes and averages 45 people per trip. It is estimated that for every $2 <u>increase</u> in the fare, one less passenger will take the bus.

a)Represent the number of passengers riding the bus as a function of the fare charged.

let n = # of passengers

let f = fee.

$n = (\text{Current}) - (\text{decrease})$

$n = 45 - 1\left(\frac{f-50}{2}\right)$

$n = 45 - \left(\frac{1f-50}{2}\right)$

$n = 45 - 0.5f + 25$

$n = 70 - 0.5f.$

b) Represent the revenue as a function of the fare charged.

$R = (\text{fee}) * (\text{\# of passengers})$

$R = f(70 - 0.5f)$

$R = 70f - 0.5f^2$

c) Graph the function. What fare will produce the maximum revenue? What is the maximum revenue?

X:[0, 140, 10]

Y:[0, 2600, 100]

$70 fare produces max. revenue $2450.

d) What fares will produce a revenue of $2400 or greater?

$60 – $80

Pg. 105 3–6, 8, 9