Day 7: Max/Min Problems

If we look at the equation $y = 2(x-3)^2 + 4$
the vertex is (3, 4) and the graph is concave up, so the vertex has a minimum value of 4 at x = 3. The 'q' value is the max or min value, whereas the 'p' value is at which point the max or min occurs.

Examples:

1) Two numbers have a sum of 24. Their product is a maximum. What are the numbers?

First #: x

$x + ? = 24$

Second #: $24-x$

$? = 24-x$

$y = x(24-x)$

$y = 24x - x^2$

$y = -x^2 + 24x$

$y = -1(x^2 - 24x + 144 - 144)$

-12

$y = -1(x-12)^2 + 144$

V: $(12, 144)$ — x, y

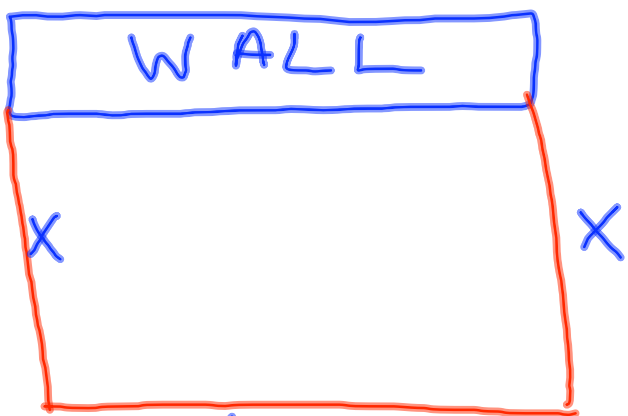
1st #: $x = 12$

2nd #: $24-x$

$24-12 = 12$

The numbers are both 12.

2) A rectangular playground is bounded on one side by a wall and the other three sides by 60 m of fencing. Determine the dimensions of the largest possible playground. (Area)



$60-2x$

$60-2(15)$

30

$A = l \times w$

$A = (60-2x)(x)$

$A = 60x - 2x^2$

$A = -2x^2 + 60x$

$A = -2(x^2 - 30x + 225 - 225)$

-15

$A = -2(x-15)^2 + 450$

$(15, 450)$

x

$x = 15$

The length is 30 m and the width is 15 m.

3) A theatre company currently charges $12 a ticket. At this price, 450 people attend each show. For every $2 increase in the price, 25 fewer people attend the show. What is the maximum revenue that the theatre company can generate?

n – # of people
f – fee

n = current – decrease

$$n = 450 - 25\left(\frac{f-12}{2}\right)$$

$$n = 450 - \frac{25f}{2} + \frac{300}{2}$$

$$n = 450 - 12.5f + 150$$

$$n = 600 - 12.5f$$

R = (# of people) × (fee)

$$R = (600 - 12.5f)(f)$$

$$R = -12.5f^2 + 600f$$

$$R = -12.5(f^2 - 48f + 576 - 576)$$

$$-24$$

$$R = -12.5(f-24)^2 + 7200$$

V: (24, 7200)

The max. revenue is $7200. When the ticket price is $24.

Assignment

Pg. 130 #3, 5, 8, 11 ~~and Handout~~