3.3 Relating Polynomial Functions and Equations

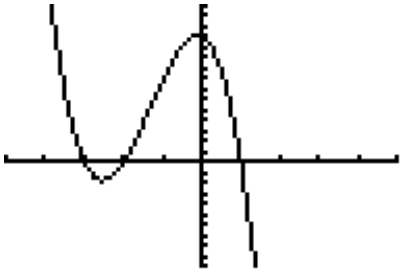
Important information

- The zeros and roots of a polynomial function are other terms used to describe the x-intercepts of the graph of the polynomial function.
- The number of zeros is equal to the degree of the polynomial
- Many different polynomials can have the same x-intercepts but be different equations

Example #1:

$y = (x-1)(x+2)(x+3)$

$y = -2(x-1)(x+2)(x+3)$



The above two functions have the same x - intercepts but a different leading coefficient. It is the leading coefficient that effects the graph of the function.

What is another equation that has the same x - intercepts as the function above??

$y = 5(x-1)(x+2)(x+3)$

$y = -4(x-1)(x+2)(x+3)$

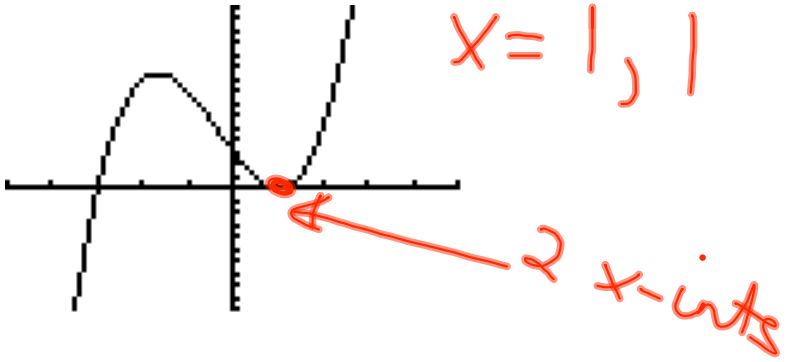
Determining the equation of a polynomial function from its graph

When you determine the equation of a graph you need to determine the following information to assist in forming the equation.

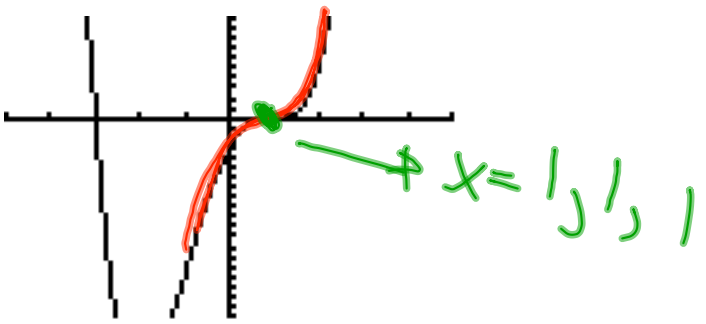
- *Read the x - intercepts from the graph*
- *Determine the coordinates of the y - intercept or a point that lies on the graph. This point CANNOT be a x-intercept.*
- *Use the x-intercepts to form factors of the equation.*
- *Use the point to determine the leading coefficient of the equation.*
- *Check your equation by graphing it on your calculator to see if you obtain the same graph as the question.*

Multiplicity of Zeros

When a graph bounces off of the x -axis
It means there are 2 zeros at that point.

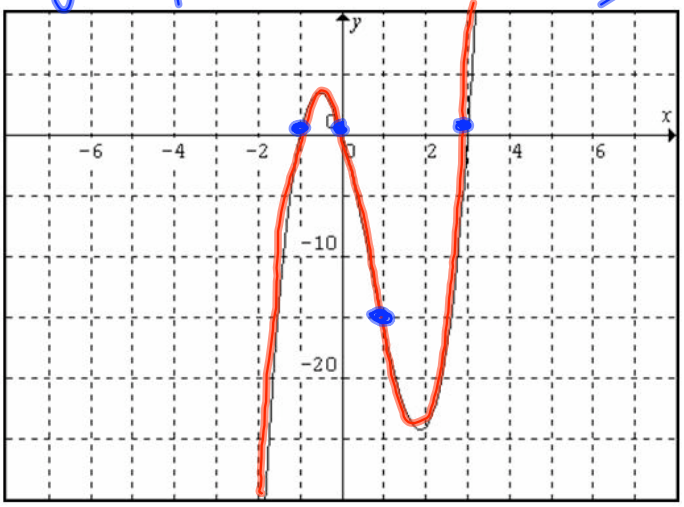


When the graph passes through the x-axis in an S-shape this means there are 3 zeros at that point.



Determine the equation of the following graphs

$y = \frac{15}{4}(x+1)(x)(x-3)$



$x = -1, 0, 3$

pt $(1, -15)$

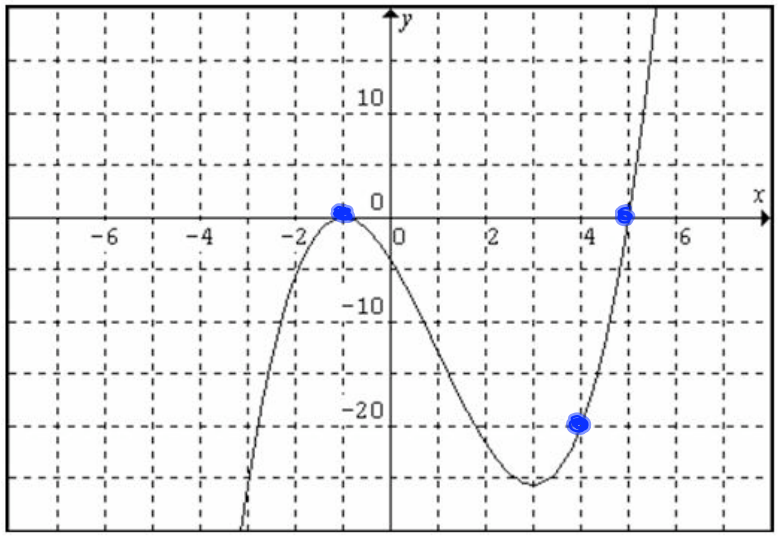
x y

$y = a(x+1)(x)(x-3)$

$-15 = a(1+1)(1)(1-3)$

$-15 = -4a$

$\frac{15}{4} = a$



$x = -1, -1, 5$

$y = a(x+1)(x+1)(x-5)$

$y = a(x+1)^2(x-5)$

$(4, -20)$

$-20 = a(4+1)^2(4-5)$

$-20 = -25a$

$\frac{20}{25} = a$

$\frac{4}{5} = a$

$y = \frac{4}{5}(x+1)^2(x-5)$

$\rightarrow x = -1 \qquad (x+1)$

$\rightarrow x = 0 \qquad (x)$

$\rightarrow x = 3 \qquad (x-3)$

Example #3:

Determine the equation of the polynomial that has zeros -1 and 4 and whose graph passes through the point (-2,2)

$y = a(x+1)(x-4)$

$2 = a(-2+1)(-2-4)$

$2 = 6a$

$a = 2/6 = \frac{1}{3}$

$y = \frac{1}{3}(x+1)(x-4)$

Sketching Graphs of Polynomials

You need to do the following when you graph a polynomial function

- *Determine and plot the y-intercept*
- *Determine and plot the x-intercepts*
- *Examine the degree of the polynomial and leading coefficient to determine in what quadrants the graph starts and ends.*
- *Examine any multiplicity of zeros to determine where the graph touches the x-axis and crosses the x-axis.*

Example #4:

Sketch the graph of the following polynomial functions

a) $y = x(x+2)(x-3)$

x=0, -2, 3

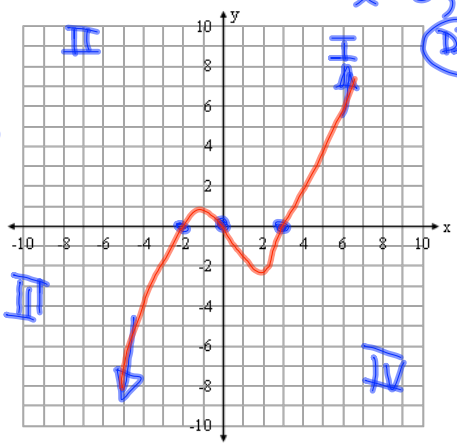
Degree: 3

x=0

y-int:

$y = 0(0+2)(0-3)$

$y = 0$



b) $y = (x+1)^2(x-2)^2$

x = -1, -1, 2, 2

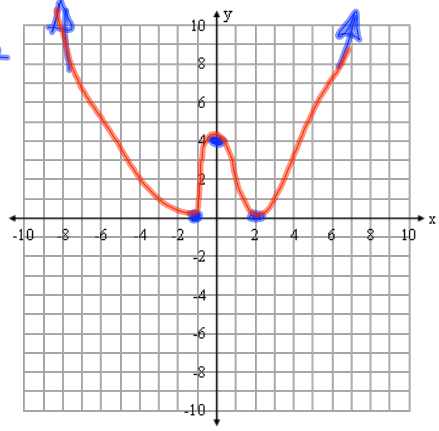
y-int: x=0

$y = (0+1)^2(0-2)^2$

$1 \cdot 4$

$y = 4$

Degree: 4



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$x(x+2)(x-3)$

$x = 0$

$x+2=0 \rightarrow x=-2$

$x-3=0 \rightarrow x=3$