

3.9 Composition of Functions

Function Operations

- **Adding functions** is expressed as $f(x) + g(x)$
- **Subtracting functions** is expressed as $f(x) - g(x)$
- **Multiplying functions** is expressed as $f(x) \cdot g(x)$

Adding Functions

When you add two functions you combine like terms to form one single function.

$$\text{If } f(x) = \underline{2x^2 + 5x + 3} \text{ and } g(x) = \underline{-4x^2 - 7x + 6}$$

Then $f(x) + g(x)$ is

$$\begin{array}{r} (2x^2 + 5x + 3) + (-4x^2 - 7x + 6) \\ \hline -2x^2 - 2x + 9 \end{array}$$

Subtracting Functions

When you subtract two functions you combine like terms to form one single function.

$$\text{If } f(x) = \underline{2x^2 + 5x + 3} \text{ and } g(x) = \underline{-4x^2 - 7x + 6}$$

Note: Do not forget to change the signs of all of the terms in the second function. The subtraction sign affects all of those terms.

Then $f(x) - g(x)$

$$\begin{array}{r} (2x^2 + 5x + 3) - (-4x^2 - 7x + 6) \\ \hline (2x^2 + 5x + 3) + (+4x^2 + 7x - 6) \\ \hline 6x^2 + 12x - 3 \end{array}$$

Multiplying Functions

If $f(x) = 3$ and $g(x) = 2x - 7$

then $f(x) \times g(x)$

$$\begin{array}{r} 3 \quad (2x - 7) \\ \hline 6x - 21 \end{array}$$

If you wish to find $4g(x)$

$$\begin{array}{r} 4 \cdot (2x - 7) \\ \hline 8x - 28 \end{array}$$

If $f(x) = 2x - 3$ and $g(x) = 3x + 4$

Then $f(x) \times g(x)$

$$\begin{array}{r} (2x - 3) (3x + 4) \\ \hline 6x^2 - x - 12 \end{array}$$

Composition of Functions

Composition of functions is a way of combining operations on functions.
Let's first review some notation and operations.

If $f(x) = 5x - 4$ then find $f(2)$ Remember this is read as "f" of 2.

$$f(2) = 5(2) - 4$$

$$f(2) = 6$$

Point

(2, 6)

When we work with composition of functions we work with more complex operations for example

If $f(x) = 2x + 1$ and $g(x) = 2x^2 + 3x$

Determine $f(g(2))$ Note: This is read as "f" of "g" of 2.

$f(g(2))$ ← Work from the inside to the outside

$$g(2) = 2(2)^2 + 3(2)$$

$$g(2) = 8 + 6$$

$$g(2) = 14$$

$$f(g(2))$$

$$f(14) = 2(14) + 1$$

$$f(14) = 29$$

Now let's evaluate $g(f(2))$

If $f(x) = 2x + 1$ and $g(x) = 2x^2 + 3x$

$$f(2) = 2(2) + 1$$

$$f(2) = 5$$

$$\rightarrow g(5) = 2(5)^2 + 3(5)$$

$$g(5) = 50 + 15$$

$$g(5) = 65$$

$$\begin{array}{l} \cancel{2x^2 + 3x(5)} \\ \cancel{2(2^2) + 3(2)^5} \end{array}$$

Try this one on your own

If $f(x) = 4x - 3$ and $g(x) = -3x^2 + 2x$ determine $f(g(3))$

Now let's express $f(g(x))$

If $f(x) = 2x + 1$ and $g(x) = 2x^2 + 3x$

$F(g(x))$ means everywhere there is an "x" in the function $f(x)$ we replace it with the function $g(x)$.

$$f(g(x)) = 2(2x^2 + 3x) + 1$$

$$4x^2 + 6x + 1$$

Now let's express $g(f(x))$ as functions of x .

$$g(f(x)) = 2(2x+1)^2 + 3(2x+1)$$

$$= 2 \cdot (2x+1)(2x+1) + 3 \cdot (2x+1)$$

$$2 \cdot (4x^2 + 4x + 1) + 6x + 3$$

$$8x^2 + 14x + 5$$

Try this one on your own

If $f(x) = 3x - 4$ and $g(x) = 4x^2 - 3x$

Find $f(g(x))$

$$\begin{aligned} f(x) &= 3x - 4 \\ f(4x^2 - 3x) &= 3x - 4 \\ &= 3(4x^2 - 3x) - 4 \\ &= 12x^2 - 9x - 4 \end{aligned}$$

$$f(x) = 2x + 1 \quad \leftarrow$$

$$f(x) = x^2 - 5$$

Assignment: Pg. 188 1-6 odds
Pg. 217 1-3, 5, 9, 10