Day 3 - The Remainder Theorem

Recall long division from middle school!!!

Divisor

Quotient

$$\begin{array}{r} 82 \\ 2\overline{)165} \\ -\underline{16} \\ 05 \\ -\underline{4} \\ 1 \end{array}$$

Dividend

Remainder

Dividend = (divisor)(quotient) + remainder

$$165 = \underbrace{2(82) + 1}_{\text{Answer}}$$

Well, you did long division on this too in Pure 10!

$$\left(x^3 - 3x^2 + 3x - 1\right) \div (x - 2)$$

$$
\begin{array}{r}
x^2 - 1x + 1 \\
(x-2)\overline{\big)\, x^3 - 3x^2 + 3x - 1} \\
-\ \underline{x^3 - 2x^2 } \\
-1x^2 + 3x \\
-\ \underline{-1x^2 + 2x } \\
1x - 1 \\
-\ \underline{1x - 2} \\
1
\end{array}
$$

$$\underline{(x-2)\left(x^2 - 1x + 1\right) + 1}$$

There is a shorthand that exists called synthetic division, of which we will use the subtraction method:

$$\left(x^3 - 3x^2 + 3x - 1\right) \div \left(x - 2\right)$$

$$\begin{array}{r|rrrr}
-2 & 1 & -3 & 3 & -1 \\
 & \downarrow & - \,{-2} & - \,2 & - \,{-2} \\
\hline
 & 1 & -1 & 1 & \boxed{1}
\end{array}$$

$$(x-2)(1x^2 - 1x + 1) + 1$$

Try it on this one:

$$\left(x^3+4x^2+x-6\right)\div\left(x-2\right)$$

$$\begin{array}{r|rrrr} -2 & 1 & 4 & 1 & -6 \\ & \downarrow & -2 & -12 & -26 \\ \hline & 1 & 6 & 13 & 20 \end{array}$$

$$(x-2)\left(1x^2+6x+13\right)+20$$

$$\left(2x^3+5x^2-x-15\right)\div\left(x-3\right)$$

$$\begin{array}{r|rrrr} -3 & 2 & 5 & -1 & -15 \\ & \downarrow & -6 & -33 & -96 \\ \hline & 2 & 11 & 32 & 81 \end{array}$$

$$(x-3)\left(2x^2+11x+32\right)+81$$

The remainder theorem:

When a polynomial in x is divided by x - k, the remainder is equal to the number obtained by substituting k for x in the polynomial.

Determine the remainder when

$$\left(x^3+4x^2+x-6\right)\div(x-2)$$

$x-2=0$

$x=2$

$(2)^3+4(2)^2+(2)-6$

$8+16+2-6$

Remainder $\rightarrow$ 20

Do the same for:

$$\underbrace{\left(x^3+4x^2+x-6\right)}\div\cancel{(x+1)}$$

$x=-1$

$(-1)^3+4(-1)^2+(-1)-6$

$-1+4-1-6$

-4

When $x^3 + 5x^2 + kx - 8$ is divided by $(x - 3)$ the remainder is 1. Find k.

$x^3 + 5x^2 + kx - 8 = 1$

Substitute $x = 3$

$3^3 + 5(3)^2 + k(3) - 8 = 1$

$27 + 45 + 3k - 8 = 1$

$72 + 3k - 8 = 1$

-64

$64 + 3k = 1$ -64

$\frac{3k}{3} = \frac{-63}{3}$

$k = -21$

Assignment:
Pg. 254 #1-3 odds, 5, 7